RAN - 2203000205023005

RAN-2203000205023005

T. Y. B. Sc. (Mathematics) (Sem. - V) Examination March - 2023

MTH - 505 : Mathematics

Graph Theory

[Total Marks: 50

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T. Y. B. Sc. (Mathematics) (Sem. - V)**

Name of the Subject :

☛ **MTH - 505 : Mathematics Graph Theory**

Subject Code No.: **2203000205023005**

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Follow usual notations.

(4) Figures to the right indicate marks of the question.

Que:1 Answer the following : (Any FIVE) **[10]**

(1) Find the degree of each vertex in null graph with 2022 vertices.

(2) What do you mean by isolated vertex and pendant vertex?

(3) Draw the graphs of the chemical compounds: N_2O_3 and C_6H_6.

(4) Write two differences between path and circuit.

(5) Draw a graph that has Hamiltonian path but not a Hamiltonian circuit.

(6) What is the maximum number of edges in a simple graph with 7 vertices and 2 components?

(7) Draw four different rooted trees each with four vertices.

(8) Find the maximum level of a binary tree with 15 vertices.

Que:2 Answer the following : (Any TWO) **[10]**

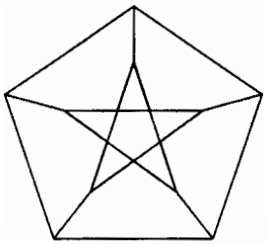
(1) State and solve Utility problem of three houses and three utilities namely, water, gas and electricity.

(2) Prove that the number of vertices of odd degree in a graph is always even.

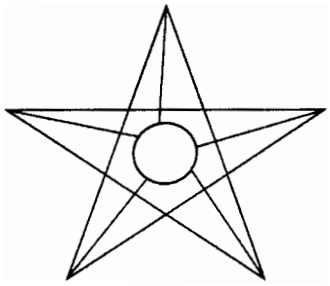
(3) Prove that in a simple graph with n vertices, the maximum degree of any vertex is $(n-1)$ and the maximum number of edges is $\frac{n(n-1)}{2}$.

Que:3 Answer the following : (Any TWO) **[10]**

(1) State necessary conditions for two graphs to be isomorphic. Are the following graphs G_1 and G_2 isomorphic? Justify your answer.



Graph G_1



Graph G_2

(2) If a graph G has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices.

(3) Prove that a connected graph G remains connected after removing an edge e_i from the graph G, if and only if e_i is in some circuit in G.

Que:4 Answer the following : (Any TWO) **[10]**

(1) Prove that the graph G with n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices v_i and v_j in G satisfy the condition $d(v_i) + d(v_j) \geq (n-1)$.

(2) State and solve seating problem about different arrangement of nine members.

(3) Prove that a graph containing m edges namely, $e_1, e_2, ..., e_m$, can be decomposed in $(2^{m-1} - 1)$ different ways into pairs of subgraphs g_1 and g_2.

Que:5 Answer the following : (Any TWO) **[10]**

(1) If the total number of vertices in a Binary tree is n,then prove that the number of internal vertices in it is $(n-1)/2$.

(2) Prove that every tree has either one or two centers.

(3) Let T be a tree with at least two vertices then prove that it has at least two pendent vertices.
